

Spectral equivalence as a proposed link between classical trajectories and quantum processes

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Abstract

Can a classical charged-particle trajectory be associated with a quantum process through a shared radiation spectrum? This Perspective examines spectral equivalence as a possible basis for such an association. For a prescribed trajectory, the Liénard–Wiechert potentials provide a classical route to the electromagnetic fields and, through Fourier analysis of the radiation field, a corresponding spectrum. The proposed inverse procedure would adjust parameters within a chosen family of trajectories to reproduce a target spectrum, including one associated with a quantum process. We outline this framework and distinguish the reported numerical field calculations from the spectral fitting that remains to be demonstrated in this article. No fitted trajectory or quantitative spectral comparison is presented here. A spectral match would establish correspondence for the specified observable and comparison conditions; it would not, by itself, establish a unique particle path or a complete account of quantum dynamics. We identify questions concerning existence, uniqueness, physical consistency, and validation that must be addressed before assigning broader significance to equivalent-spectrum trajectories. The immediate objective is to formulate a testable research programme whose conclusions remain proportionate to the evidence obtained.

KEYWORDS

Quantum foundations; Classical electrodynamics; Particle trajectories; Radiation spectra; Spectral equivalence; Inverse modelling

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The Question Posed by Spectral Equivalence

Atomic emission spectra provided an important motivation for the development of quantum theory. In classical electrodynamics, an accelerating charge emits radiation [1]. Applying this description to an orbiting atomic electron raises the problem of energy loss and atomic stability, which featured in the development of early atomic models [2]. Bohr–Sommerfeld approaches retained orbital descriptions subject to quantization conditions [2,3], whereas matrix mechanics established a formulation that did not require a classical electron trajectory [4]. These developments provide the background for asking whether a limited correspondence between classical trajectories and quantum observables can nevertheless be constructed.

Our proposal concerns one such observable: the radiation spectrum. Given a classical trajectory, electromagnetic theory provides a forward calculation of the emitted radiation. The inverse question is whether a trajectory can be found whose calculated spectrum reproduces a specified target. When that target is associated with a quantum process, a successful match would define a spectral correspondence between the two descriptions. Establishing the existence and limits of this correspondence is the central task considered here.

The use of trajectories in quantum theory already has a substantial foundation in Bohmian mechanics [5-7]. In that framework, particle motion is specified through a guidance law associated with the quantum state. The spectral approach considered here would infer candidate trajectories by matching a selected observable. It does not presently supply a guidance law, a measurement theory, or an account of entangled systems. Consequently, its value must first be assessed at the level of the proposed spectral correspondence.

Proposed Spectral Equivalence Framework

For a point charge q moving along a prescribed trajectory $\mathbf{r}_0(t)$, the scalar and vector Liénard–Wiechert potentials can be written as [1]

$$\varphi(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(1-n\cdot\beta)R} \right]_{\text{ret}} \quad (1)$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \left[\frac{q\mathbf{v}}{(1-n\cdot\beta)R} \right]_{\text{ret}} \quad (2)$$

Here, v is the charge velocity, $\beta = v/c$, R is the distance from the retarded source position to the observation point, and $\mathbf{n}$ is the corresponding unit vector. The subscript *ret* indicates evaluation at the retarded time. The electric and magnetic fields follow from

$$\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t} \quad (3)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (4)$$

The retarded time is determined implicitly by

$$t_{\text{ret}} = t - \frac{|\mathbf{r} - \mathbf{r}_0(t_{\text{ret}})|}{c} \quad (5)$$

For general trajectories, evaluating this relation requires a numerical procedure. We developed programs in C++ and Python to solve the retarded-time equation over a grid of positions and times and used those solutions to calculate the electromagnetic fields and the associated spectral quantity. The code is available from the authors upon request. These implementation steps do not establish that a trajectory reproducing a particular quantum-process spectrum has been obtained; no such fitted example or numerical validation is presented in this Perspective.

The relevant field for the spectral comparison is the radiation contribution. In the radiation zone, its magnetic field is given by the cross product of the propagation direction and the radiation electric field, divided by c [1]. At a fixed observation point, the Fourier transform can be expressed formally as

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$$\tilde{E}_{rad}(\omega) = \int_{-\infty}^{+\infty} E_{rad}(t) e^{i\omega t} dt \quad (6)$$

A corresponding spectral quantity is proportional to the squared magnitude of this transform:

$$S(\omega) \propto |\tilde{E}_{rad}(\omega)|^2 \quad (7)$$

The symbol S is used here without assigning a universal power normalization. The observation geometry, integration interval, and normalization must be specified before comparing it with a target spectrum. A finite-duration Fourier transform naturally describes a spectral energy distribution; a power spectrum additionally requires an appropriate time normalization. A spectrum at one observation direction must also be distinguished from radiation integrated over directions [1]. These choices are part of the definition of the proposed comparison.

The inverse step would begin with a target spectrum $F(\omega)$ and a family of trajectories $r_0(t; \alpha_1, \dots, \alpha_n)$. For each candidate parameter set, the forward calculation would produce a spectrum $S(\omega; \alpha_1, \dots, \alpha_n)$. The parameters would then be varied to reduce the discrepancy between the calculated and target spectra. The fitting criterion, frequency interval, normalization, and acceptable discrepancy would have to be specified in advance. This procedure could be applied to a target of quantum-mechanical origin, but its success for any particular target remains to be demonstrated here.

Scope and Limits of the Proposed Correspondence

We use “equivalent-spectrum trajectory” to mean a trajectory whose calculated radiation spectrum agrees with a specified target under explicitly stated comparison conditions. In a numerical study, any equivalence would be qualified by the fitting tolerance and numerical resolution. Matching spectral peak positions alone would support a narrower conclusion than matching both positions and intensities across the comparison interval. The intended level of agreement should therefore be stated before interpreting the outcome.

Uniqueness is an open question for this proposal. Equation (7) retains the squared magnitude of the Fourier transform and does not retain its phase. A match of that quantity alone therefore cannot establish a unique field history or trajectory without further constraints. Future work should examine whether distinct trajectories within the selected family, or across different families, can reproduce the same target. If multiple solutions occur, the result would need to be described as a set of candidates rather than an identified particle path.

Physical consistency presents a separate requirement. Prescribing a trajectory and calculating its radiation does not show that the trajectory follows from a physically specified force law or energy balance. An application to atomic transitions would need to address how the proposed motion is sustained, how radiated energy is accounted for, and how the description relates to the initial and final atomic states. These questions remain unresolved by spectral fitting alone. In particular, the framework presented here does not resolve the classical problem of atomic stability.

The implications for quantum foundations must likewise remain limited. Questions about measurement and the completeness of quantum descriptions have a history extending beyond spectral observations [8,9]. Bell-type experiments concern correlations that are not tested by fitting a single radiation spectrum [10]. No analysis of those correlations or of relativistic compatibility is supplied by this framework. It would therefore be premature to claim that spectral equivalence resolves the measurement problem or avoids the foundational difficulties discussed in connection with trajectory theories.

A Research Agenda for Evaluating the Proposal

A first evaluation should use a fully specified target and a reproducible family of trajectories. The target's source, units, observation conditions, and frequency range should be reported together with the trajectory parameters and admissibility constraints. The resulting trajectory and spectrum should be shown explicitly, with a

quantitative measure of the discrepancy. This would allow readers to assess the proposed correspondence directly.

The forward calculation should first be checked against a case with a known radiation spectrum. Numerical checks should then examine the retarded-time solution, time sampling, observation duration, and extraction of the radiation field. These checks would help distinguish a spectral correspondence from features introduced by numerical settings. They are proposed validation steps; no outcomes from such checks are reported here.

The inverse calculation should also be examined for sensitivity to initial parameter choices and to the chosen trajectory family. Failure to fit a target within one family would constrain that family rather than establish that no classical trajectory can reproduce the target. Conversely, a successful fit would require examination of alternative solutions and physical admissibility before receiving a broader interpretation.

A longer-term question is whether independently obtained equivalent-spectrum trajectories share a governing differential equation. Finding such a relation could motivate a candidate dynamical model. Its status would depend on whether it predicts additional behaviour beyond the spectra used to construct it. Merely finding a differential equation that accommodates fitted trajectories would not establish equivalence with quantum mechanics. This distinction is essential when considering the original motivation of associating trajectories with quantum processes.

Conclusion

Spectral equivalence offers a focused question for investigating possible correspondences between classical charged-particle motion and quantum-process spectra. The Liénard-Wiechert framework supplies the forward calculation, while parameter fitting defines a possible inverse route. The contribution of this Perspective is to formulate that route and identify the conditions under which a spectral match could be interpreted. Demonstrated fits, numerical validation, examination of alternative solutions, and physical consistency are required before stronger conclusions can be drawn. Any subsequent dynamical proposal would need independent predictive tests extending beyond the observables used in its construction.

Data Availability Statement

No original dataset or fitted-spectrum results are reported in this Perspective. The C++ and Python code described in the article is available from the corresponding author upon reasonable request.

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Ethics Declaration

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Conflict of Interest

The authors declare no conflict of interest.

Preprint Disclosure

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